Ch. 4: **Excess carriers In Semiconductors**

- Excess carriers: extra carriers of values that exist at thermal equilibrium

- Excess carriers can be created by many methods. In this chapter the optical absorption will be considered → we will study photoluminescence and photoconductivity

Optical Absorption (1)

Photons of selected wavelength (λ) are directed at a semiconducting sample. Absorption has a direct relation with incident photon energy *hν; If*

$h\nu \geq E_g$ Absorption

$h\nu < E_g$ Negligible Absorption

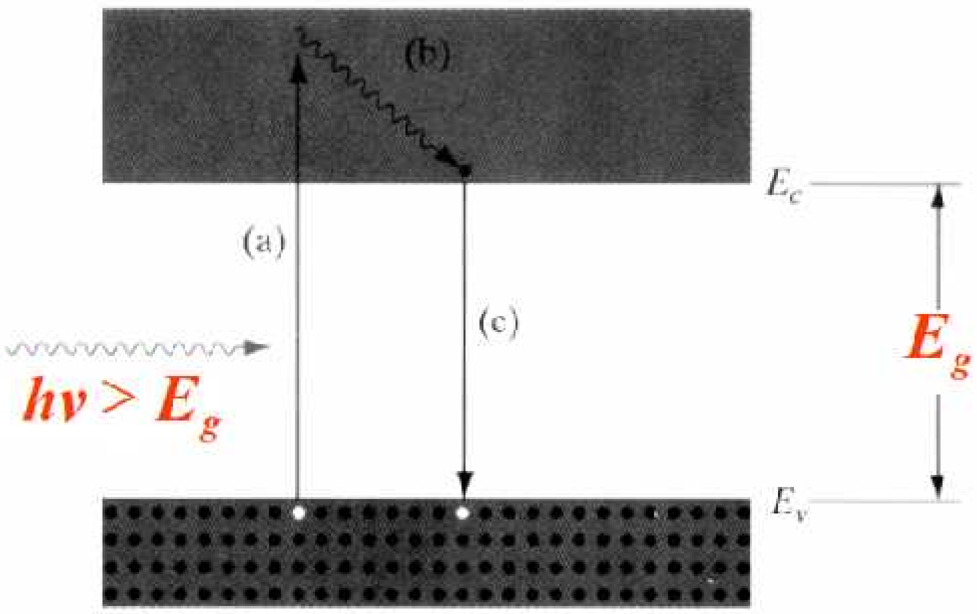
Intensity of absorbed radiation:

$$-\frac{dI(x)}{dx} = \alpha I(x)$$

➡ Transmitted intensity through depth *x* in cm is

$$I(x) = I_0 e^{-\alpha x}$$

α is absorption coefficient in cm^{-1}



When hν >E_g → the energy is absorbed by electrons→ they will exited from V.B. to C.B.

(a) An electron-hole pair (EHP) is created, (b) the excited electron gives up energy to the lattice to scattering events, (c) the electron recombines with a hole in the valence band

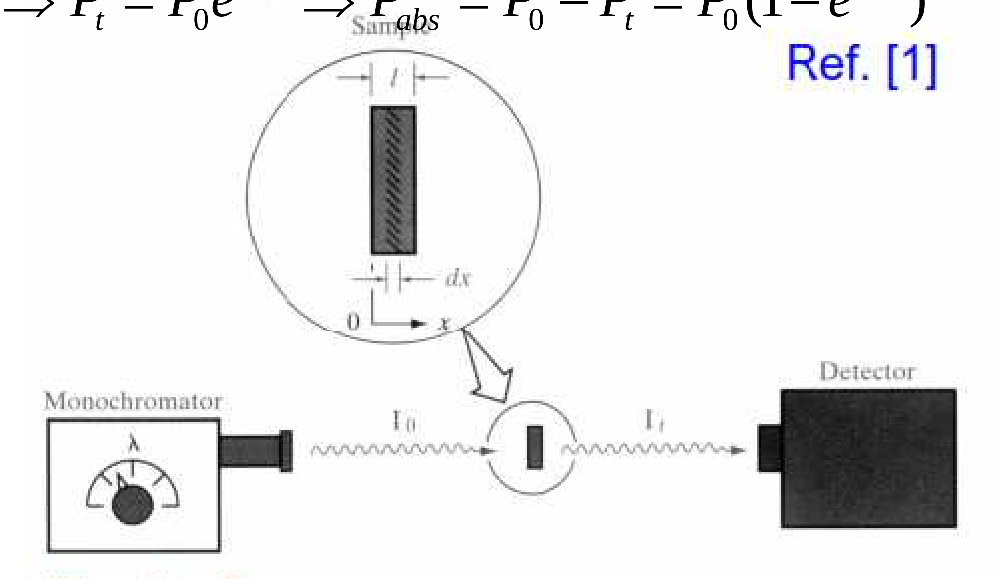
Optical Absorption (2)

For sample of thickness l

$$I_t = I_0 e^{-\alpha l} \Rightarrow I_{abs} = I_0 - I_t = I_0(1 - e^{-\alpha l})$$

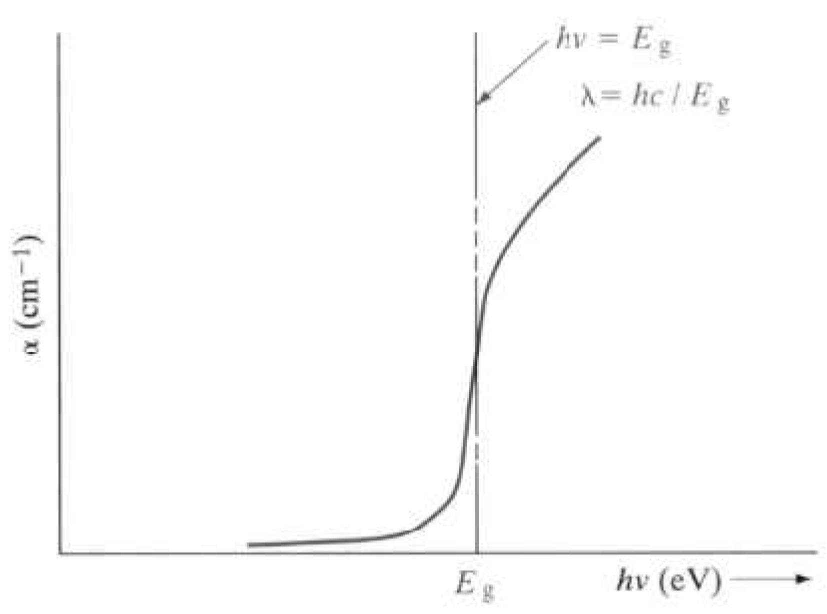
intensity of *light* is directly proportional to power

$$\Rightarrow P_t = P_0 e^{-\alpha l} \Rightarrow P_{abs} = P_0 - P_t = P_0(1 - e^{-\alpha l})$$



Ref. [1]

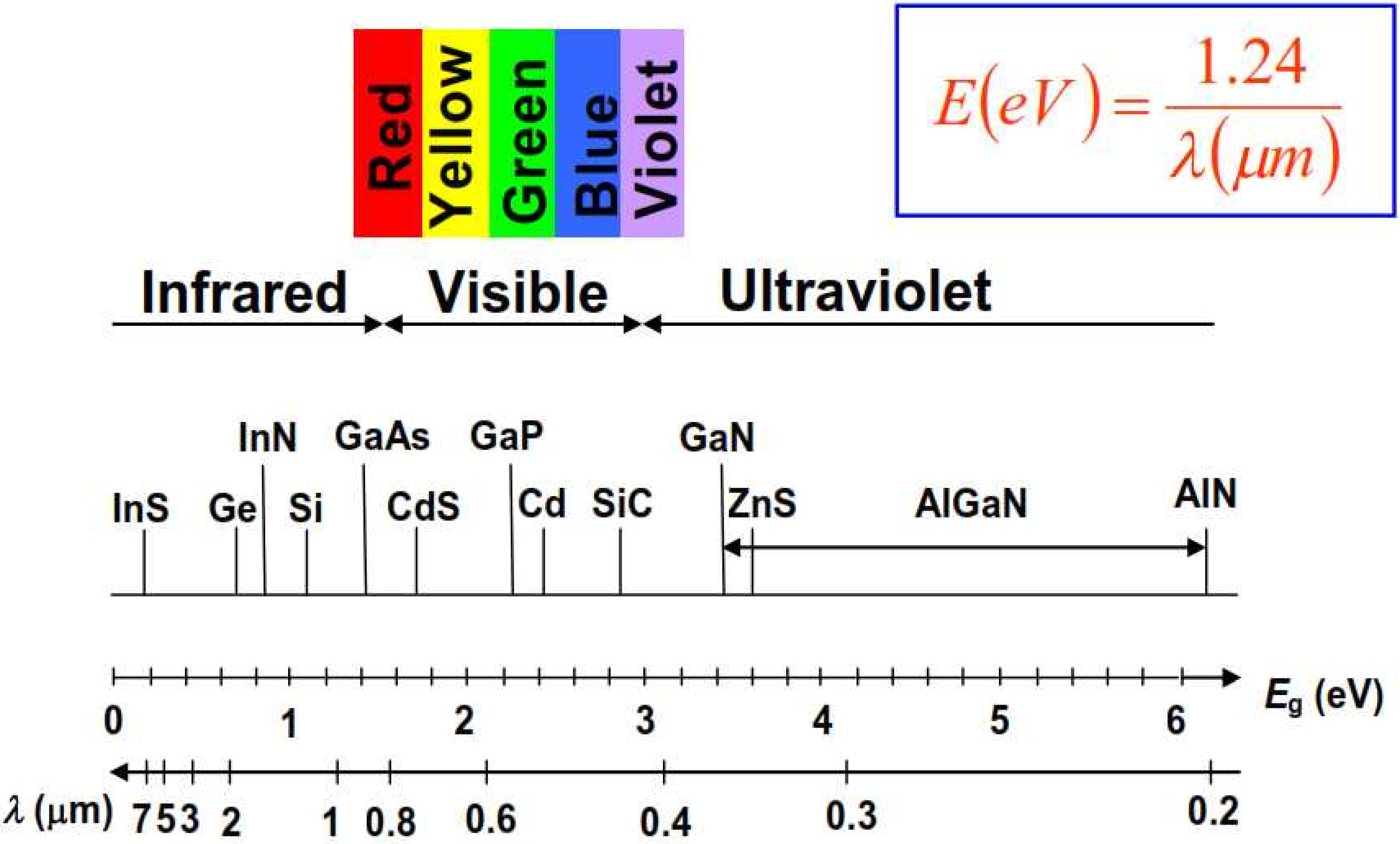
Wavelength selection



- When the energy of the incident photon is lower than bandgap, the semiconductor does not absorb radiation.
- At the band gap of semiconductor, the absorption is increased sharply and the slope of the curve is very steep.
- At higher energies, there are always states with appropriate energies that are available to make the transitions happen

Entire electromagnetic spectrum

$$E(eV) = \frac{1.24}{\lambda(\mu m)}$$


Red Yellow Green Blue Violet

Infrared Visible Ultraviolet

InN GaAs GaP GaN

InS Ge Si CdS Cd SiC ZnS AlGaN AlN

0 1 2 3 4 5 6 E_g (eV)

λ (µm) 7 5 3 2 1 0.8 0.6 0.4 0.3 0.2

Some energy gaps of some common semiconductors over the entire electromagnetic spectra

Luminescence

When exited excess carriers recombine (electrons fall from C.B. to V.B.), light can be given off the material. The general property of light emission is called luminescence. The type of luminecence depends on exitation mechanism →

Luminescence →
- Photoluminescence (e.g. Phosphorus)
- Cathodoluminescence (e.g. X-Ray)
- Electroluminescence (e.g. LED)

Photoluminescence: Carriers are generated by photon absorption. The radiation caused by the recombination of excited carriers.

Cathodoluminescence: Carriers are generated by high-energy electron bombardment of the material. The radiation caused by the recombination of excited carriers.

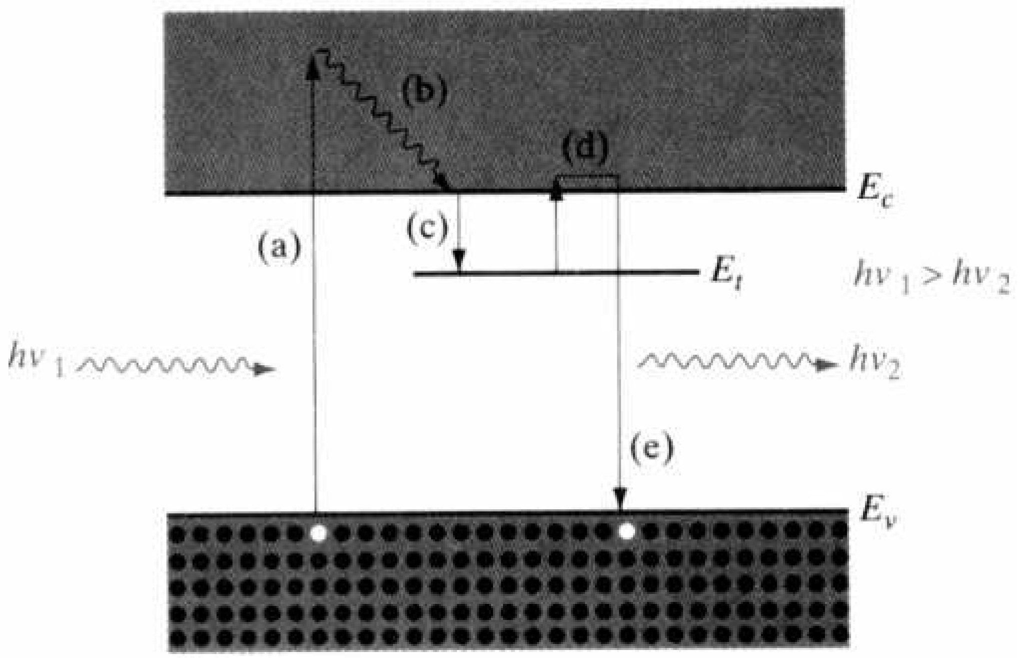
Electroluminescence: Carriers are provided by the p-type and n-type currents in the sample. The radiation is caused by the recombination of the injected carriers.

Photoluminescence

(a) Fluorescence: Direct recombination from conduction band to valence band. Fast Process, ~ 10^{-8} s . Fluorescence of direct recombination is like what shown in the figure of slide 2

(b) Phosphorescence: involved with electron trapping in a trap level in the band gap. The electron may be again excited to the conduction band, and the process may be repeated, before the electron finally recombines. Thus this is a slow process and may take seconds and even minutes.

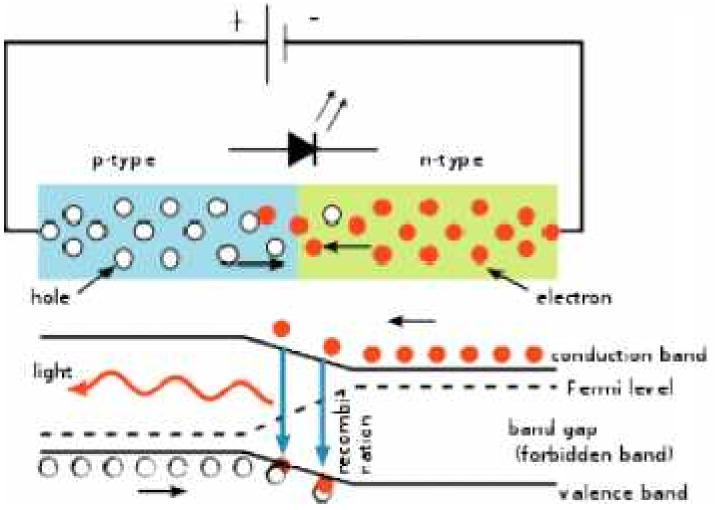
Recombination mechanism in Photoluminescence in presence of a midgap trapping level



Ref. [1]

Electroluminescence

- In this case electrical energy is used to generate photon emission in a semiconductor, e.g. in LEDs and Lasers.
- Both LEDs and lasers are based on the simple p-n junction principles. The p-region provides the holes and the n-region the electrons for recombination.
- An electric current causes the injection of electrons (or holes) into regions of the crystal where they can recombine with holes (or electrons), resulting in the emission of recombination radiation. This process is called *injection electroluminescence.*
- Remember, for efficient recombination, the material must be a <u>direct</u> bandgap material.



Example # 1

A 0.50-μm-thick sample of GaAs is illuminated with laser He-Ne beam (λ = 6328 Å). The absorption coefficient α is $4\text{x}10^4$ cm^{-1}. The laser light power incident on the sample is 5.0 mW.

(a) Find the total energy absorbed by the sample per second (J/s).

(b) Find the rate of excess thermal energy given up by the electrons to the lattice before recombination (J/s).

(c) Find the number of photons per second given off from recombination events, assuming perfect quantum efficiency.

Solution:

(a) Since $P(x) = P_0 \exp(-\alpha x)$ Then

$$P_{abs} = P_0 - P_t = P_0[1 - \exp(-\alpha d)] = 4.3\,\mathbf{mW} = 4.3\,\mathbf{mJ/s}$$

(b) One photon generates one electron which gives up to the lattice energy $\Delta E = h\nu - E_g$. This is $\Delta E / h\nu$ fraction of photon energy.

Energy of incident photon = 1.24/0.6328 = 1.96 eV
For all absorbed photons the energy transferred to the lattice per second is

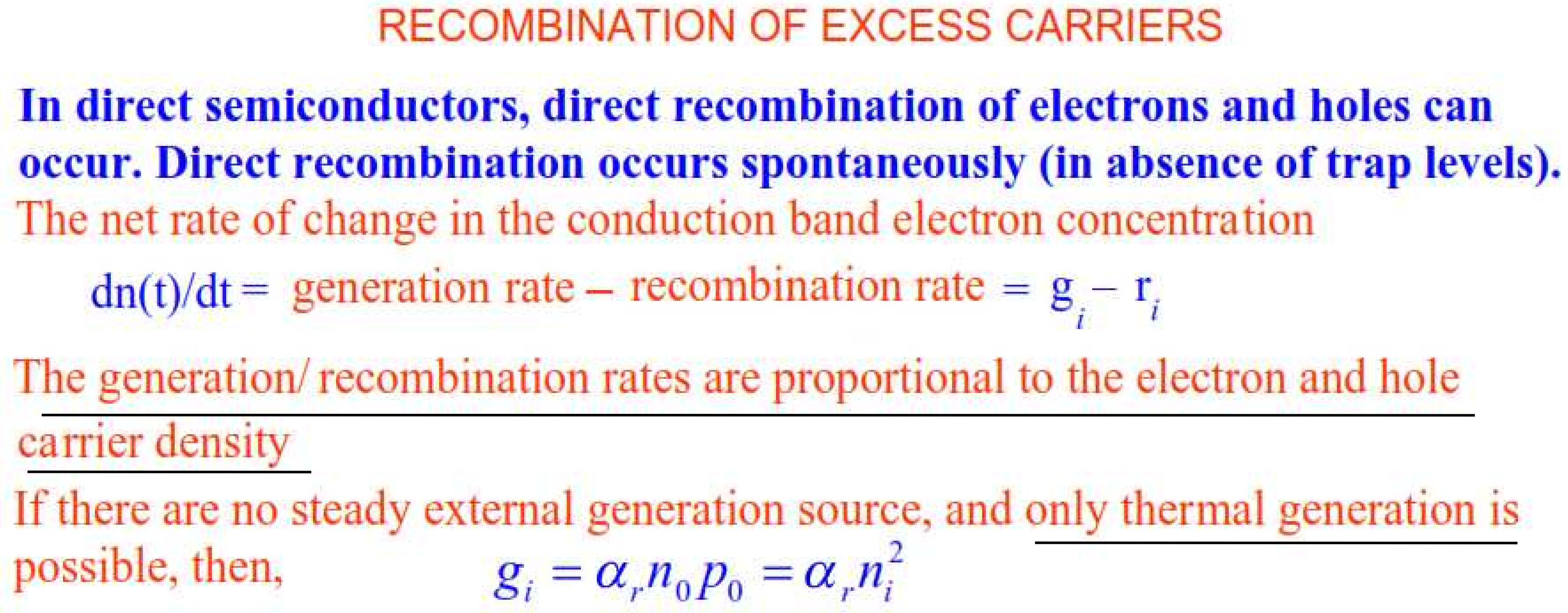
$$P_{th} = P_{abs} \cdot (h\nu - E_g)/h\nu = 4.3 \cdot (1.96 - 1.43)/1.96 = 1.16\,\mathbf{mW}$$

(d) For 100% quantum efficiency, one incident photon gives rise to one emitted photon. The total number of emitted photons (equal to the number of incident photons) per unit time is given by

$$N_{em} = \frac{P_{abs}}{h\nu} = \frac{4.3 \cdot 10^{-3}\,\mathbf{J/s}}{1.96 \cdot 1.6 \cdot 10^{-19}\,\mathbf{J}} = 1.37 \cdot 10^{16}\,\mathbf{s}^{-1}$$

Carrier life time and photoconductivity

- When excess electrons and holes are created → increase in carrier concentration → increase in conductivity
- If excess carriers arise from photoluminescence, the increase in conductivity is called photoconductivity. This appears in many photoconductive devices
- At steady state light illumination→ electron concentration becomes $n = n_0+\Delta n$ and hole concentration becomes $p = p_0 + \Delta p$.
- Δn and Δp are called steady state excess electrons and holes respectively.



RECOMBINATION OF EXCESS CARRIERS

In direct semiconductors, direct recombination of electrons and holes can occur. Direct recombination occurs spontaneously (in absence of trap levels).

The net rate of change in the conduction band electron concentration

$$dn(t)/dt = \text{generation rate} - \text{recombination rate} = g_i - r_i$$

The generation/ recombination rates are proportional to the electron and hole carrier density

If there are no steady external generation source, and only thermal generation is possible, then, $g_i = \alpha_r n_0 p_0 = \alpha_r n_i^2$

α_r is called proportionality constant

Direct Recombination

At thermal equilibrium, $r_i = g_i = \alpha_r n_0 p_0 = \alpha_r n_i^2$ →

$$dn(t)/dt = dp(t)/dt = g_i - r_i = 0$$

→ No change in carrier concentration (n_0 and p_0) and $n_0 p_0 = n_i^2$

- When Excess carriers are introduced from light absorption →

$dn(t)/dt$ =(thermal generation rate + light generation rate) - recombination rate

$dn(t)/dt = \alpha_r n_i^2$ + light generation rate – $\alpha_r np$
(Note that $n \neq n_0$ and $p \neq p_0$ and $np \neq n_i^2$)
n and p are called steady state carrier concentration

If light is turned off → light generation rate is zero

spontaneous recombination:

Excess concentration of electrons

Excess concentration of holes

$$\frac{dn(t)}{dt} = \alpha_r n_i^2 - \alpha_r n(t) p(t) \qquad n(t) = n_0 + \delta n(t) \qquad p(t) = p_0 + \delta p(t)$$

$$\Rightarrow \quad \frac{d\delta n(t)}{dt} = \alpha_r n_i^2 - \alpha_r [n_0 + \delta n(t)][p_0 + \delta p(t)] = -\alpha_r [(n_0 + p_0)\delta n(t) + \delta n^2(t)]$$

Direct Recombination

It is often we have low level of carrier creation compared to total number of charged carrier → $\delta n(t)$, $\delta p(t) << n_0 + p_0$ → $\delta n^2(t) << (n_0 + p_0)\delta n(t)$

➡ $$\frac{d\delta n(t)}{dt} = -\alpha_r (n_0 + p_0)\delta n(t)$$

Solution: $$\boxed{\delta n(t) = \Delta n \exp[-\alpha_r (n_0 + p_0)] \quad = \Delta n \exp(-t/\tau_n)}$$

$$\tau_n = \frac{1}{\alpha_r (n_0 + p_0)}$$ is called recombination lifetime or minority carrier lifetime if the injection level is low. → $\delta n(t)$ or $\delta p(t) \le 0.1(n_0+p_0)$

For n-type semiconductor: $n_0 >> p_0$ ➡ $\tau_p = (\alpha_r n_0)^{-1}$

➡ $$\boxed{\delta p(t) = \Delta p \exp(-\alpha_r n_0 t) = \Delta p \exp(-t/\tau_p)}$$

→ Decay in holes is more visible

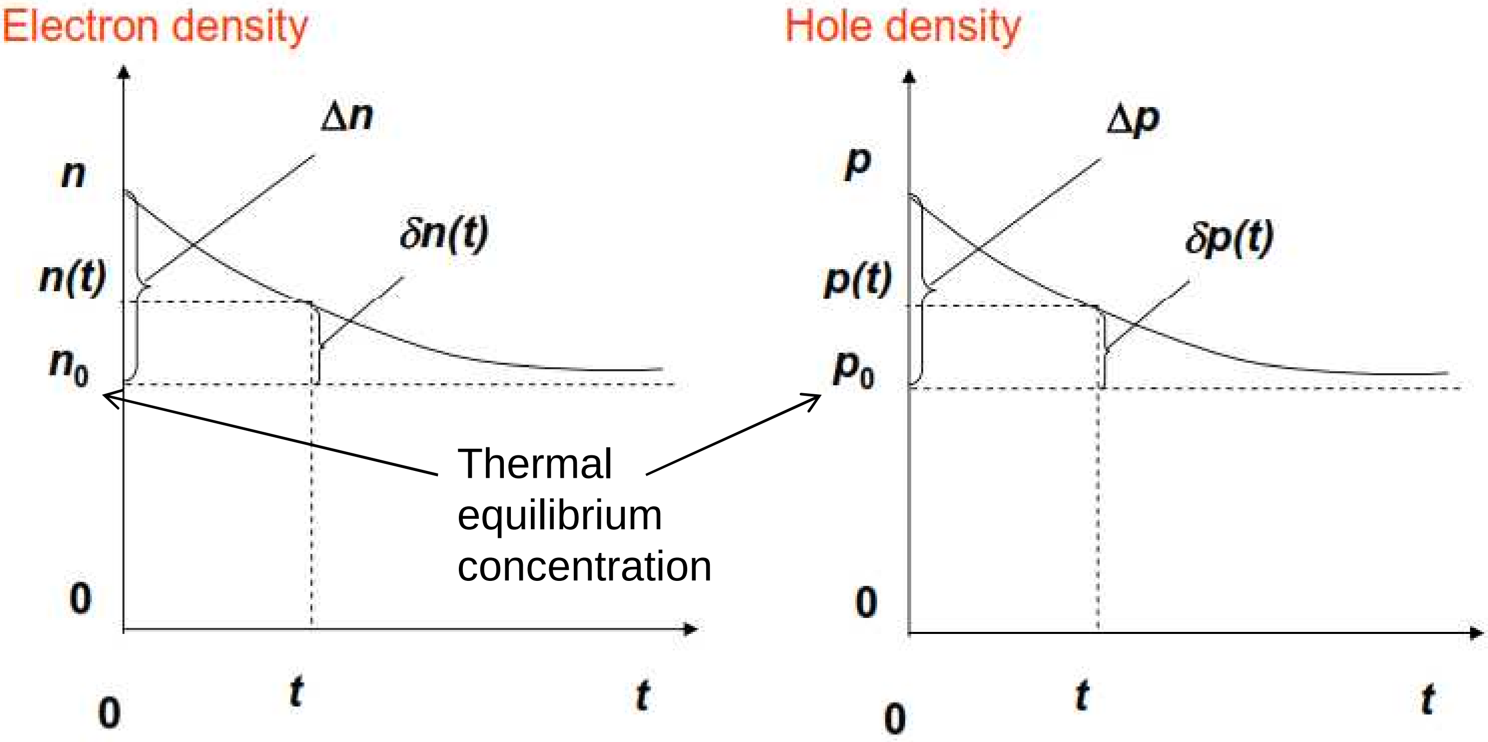
Δn and Δp are called steady state excess electrons and holes before turning off light or t = 0.

For p-type semiconductor: $p_0 >> n_0$ ➡ $\tau_n = (\alpha_r p_0)^{-1}$

➡ $$\boxed{\delta n(t) = \Delta n \exp(-\alpha_r p_0 t) = \Delta n \exp(-t/\tau_n)}$$ → Decay in electrons is more visible

In recombination process of excess carriers, we looked at minority carrier decay because it is more visible; When excess carriers are created, minority carriers have a large percentage change compared to the change in majority carrier concentration

Decay of excess carriers with time



Electron density

Hole density

Thermal equilibrium concentration

The exponential decay of excess carriers to their equilibrium value

The example below shows that the decay in minority carriers is more visible

GaAs sample doped with $N_a = 10^{15}\ \mathrm{cm}^{-3}$ of acceptors. → p-type GaAs

Thus, $p_0 = N_a = 10^{15}\ \mathrm{cm}^{-3}$.

For GaAs at $T = 300\mathrm{K}$ $n_i = p_i = 2\mathrm{x}10^{6}\ \mathrm{cm}^{-3}$.

Then $n_0 = \dfrac{n_i^2}{p_0} = 4\times10^{-3}\ \mathbf{cm}^{-3}$

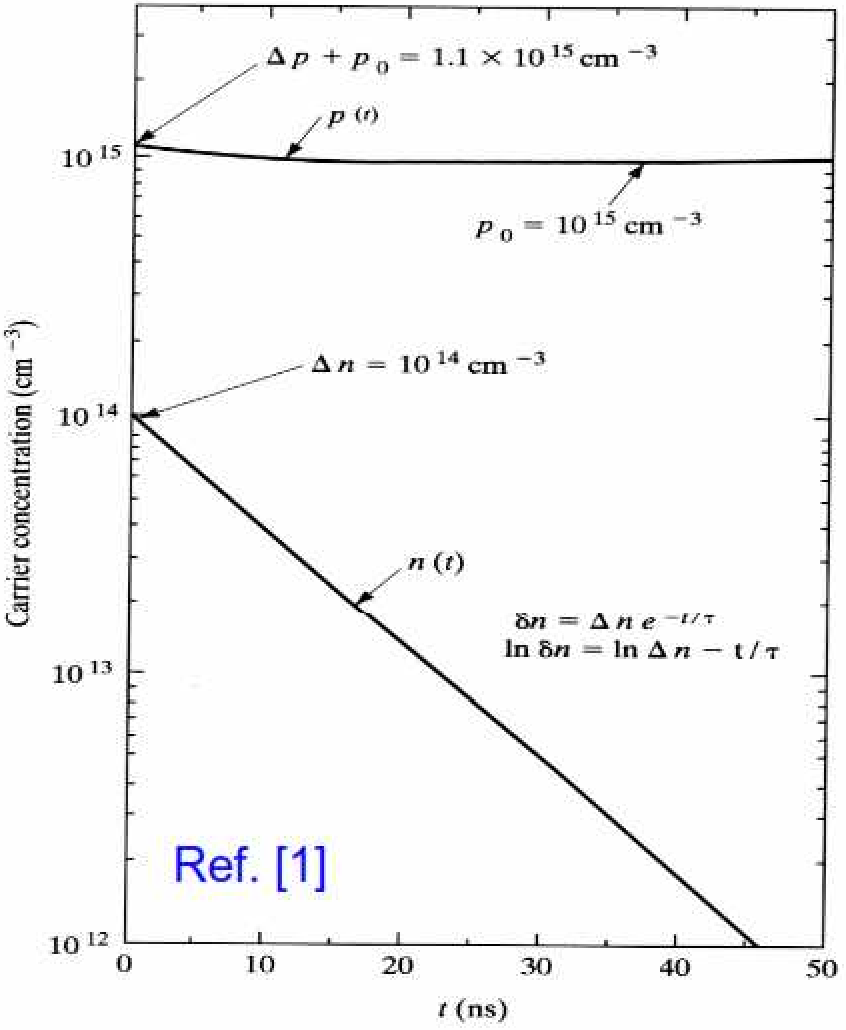
Let's assume that at $t = 0$,

$\Delta n = \Delta p = 0.1p_0 = 10^{14}\ \mathrm{cm}^{-3}$ and $\tau = 10$ ns.

→ electron concentration at steady state is $n = n_0 + \Delta n = 4\mathrm{x}10^{-3} + 10^{14} = 10^{14}\ \mathrm{cm}^{-3}$

→ hole concentration at steady state is $p = p_0 + \Delta p = 10^{15} + 10^{14} = 1.1\mathrm{x}10^{15}\ \mathrm{cm}^{-3}$

Hence, the increase in electrons (minority carriers) is much more visible than the increase in holes (majority carriers)



Ref. [1]

Then the exponential decay of $\delta n(t)$ is linear on this semi-logarithmic graph and more visible as shown

Indirect Recombination
Energy Levels inside Band Gap

- Energy Levels inside the band gap
  - Impurity (doping) levels
    - Donor levels
    - Acceptor levels
  - Defect levels
    - Recombination centers
    - Trapping centers (Trap levels)

Trapping Centers: a center that traps carriers without recombination taking place. The trapped electrons will most probably be emitted later.

Recombination Centers: a center that traps carriers which will most probably recombine with a carrier of the opposite sign captured by the trap subsequently.

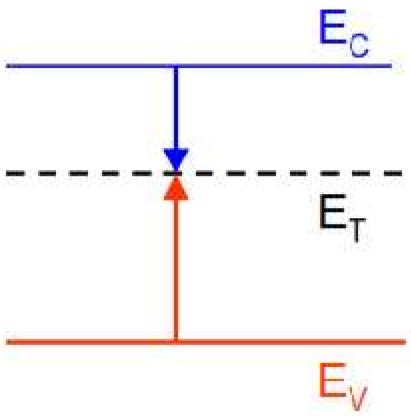
Note: It is not necessary that electron-hole recombine to produce photons. The recombination can also generate heat through creation of phonon modes in the semiconductor.

Note: Indirect recombination rate of excess carriers depends on the density of recombination centers

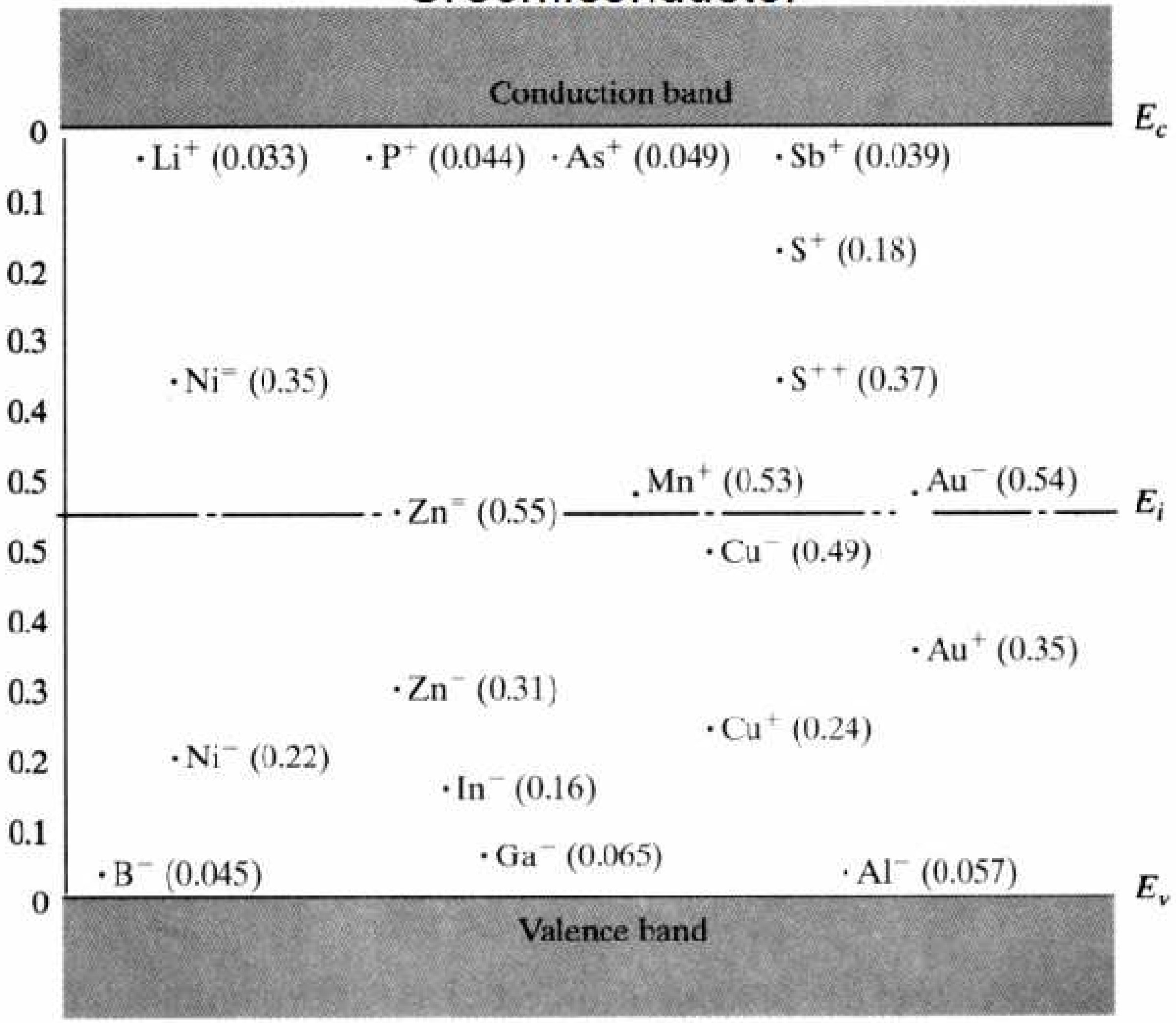
RECOMBINATION OF EXCESS CARRIERS

Indirect Recombination
Energy Levels inside Band Gap

Schematic representation of a recombination event via midgap traps



Si semiconductor



- sign indicates acceptors
+ sign indicates donors

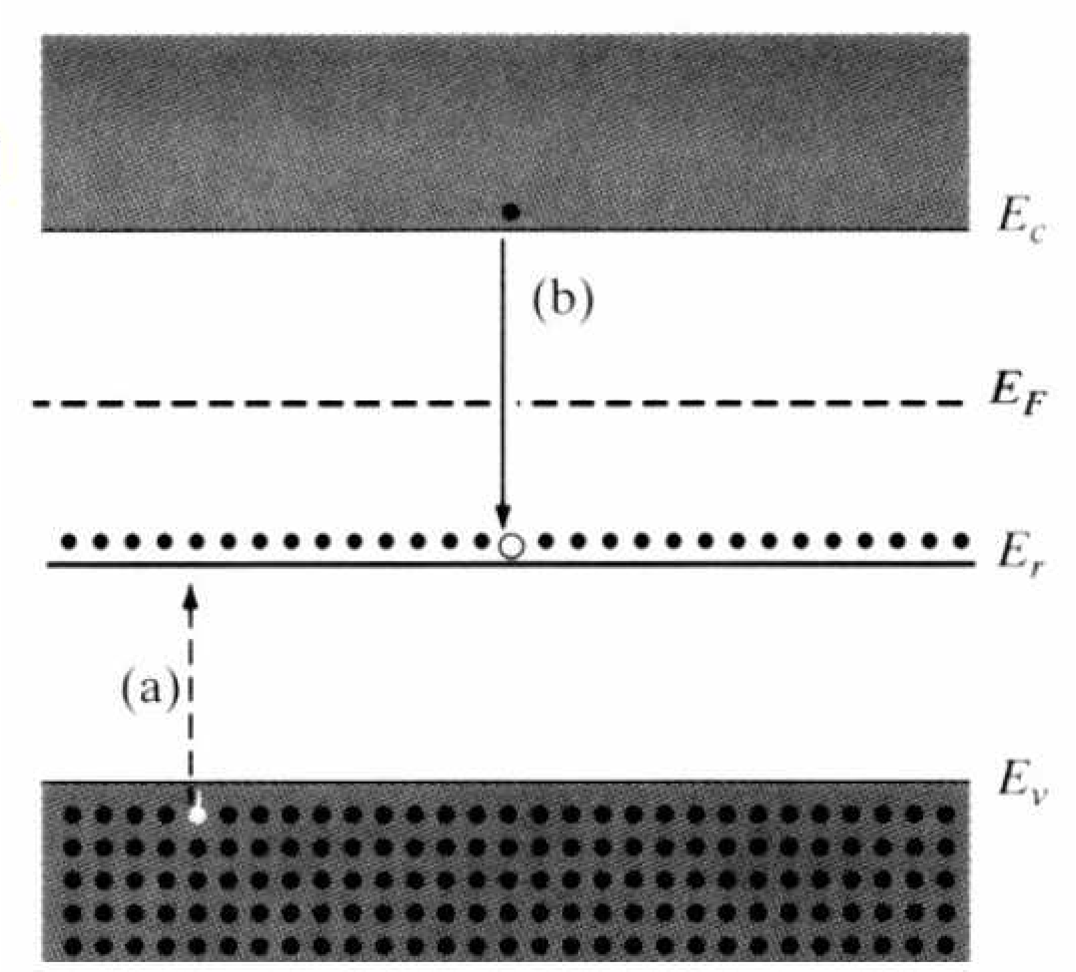
Indirect Recombination

If the recombination centers are close to the valence band

(a) Hole capture
(b) Electron capture

If the recombination centers are close to the conduction band

(a) Electron capture
(b) Hole capture



Steady State Carrier *Generation*

If a steady light is shone on the sample,

Total generation rate = $g(T)+g_{op}=\alpha_r np=\alpha_r(n_0+\delta n)(p_0+\delta p)$

where equilibrium generation rate $g(T)=\alpha_r n_0 p_0$

For steady state recombination and no trapping, $\delta n=\delta p$

➡ $$\cancel{g(T)}+g_{op}=\cancel{\alpha_r n_0 p_0}+\alpha_r[(n_0+p_0)\delta n+\delta n^2]$$

↑

For low excitation level (δn^2 is neglected)
$\delta n \le 0.1(n_0+p_0)$

➡ $$g_{op}=\alpha_r(n_0+p_0)\delta n=\frac{\delta n}{\tau_n}$$

→ Excess carrier concentration δn=Δn and δp=Δp are equal and given by

➡ $$\boxed{\begin{aligned}\delta n&=\delta p=g_{op}\tau_n\\ \Delta n&=\Delta p=g_{op}\tau_n\end{aligned}}$$

Here $\tau_n=\tau_p$ (no trapping).

When trapping is present, usually, $\tau_n\neq\tau_p$

Example

As a numerical example, let us assume that 10^{13} EHP/cm^3 are created optically every microsecond in a Si sample with $n_0 = 10^{14}$ cm^{-3} and $\tau_n = \tau_p =$ 1 μsec.

a) Find minority carrier concentration at thermal equilibrium
b) Find minority carrier concentration at steady state
c) Majority carrier concentration at steady state

Solution:

a) $$p_0 = n_i^2/n_0 = (2.25 \times 10^{20})/10^{14} = 2.25 \times 10^6 \text{ cm}^{-3} \qquad (\textit{equilibrium})$$

b) $\Delta p = g_{op}\tau_p = 10^{13}$ cm^{-3}μs^{-1} (2μs) = 2×10^{13} cm^{-3} >> p_0

→ $p = p_0 + \Delta p = 2\times10^{13}$ cm^{-3}

c) $n = n_0 + \Delta n = 1.2\times10^{14}$ cm^{-3}

Quasi-Fermi Level

Equilibrium State: $n_0 p_0 = n_i^2$

When excess carriers are present: $np \neq n_i^2$

Equilibrium State:

$$n_0 = n_i \exp[(E_F - E_i)/kT]$$

$$p_0 = n_i \exp[(E_i - E_F)/kT]$$

When excess carriers are present, use similar expressions by defining separate quasi-Fermi levels, F_n and F_p for electrons and holes (original Fermi level splits).

$$n = n_i \exp[(F_n - E_i)/kT]$$

$$p = n_i \exp[(E_i - F_p)/kT]$$

If material is n-type → $F_n \approx E_F$ at thermal equilibrium near conduction band
If material is p-type → $F_p \approx E_F$ at thermal equilibrium near valence band

Example

A Si sample with $n_0 = 10^{14}$ cm^{-3} and $\tau_n = \tau_p = 2\mu\,\mathrm{sec}$ is irradiated with a CW laser and thus every microsecond 10^{13} e-h pairs/ cm^3 are created.
What is the position of equilibrium Fermi energy, i.e. what is $E_F - E_i$?
What is a steady state concentration of electrons n and holes p under CW laser excitation?
Find the position of quasi-Fermi levels for electrons and holes, $F_n - E_i$ and $E_i - F_p$, respectively?

$n_0 = n_i \exp[(E_F - E_i)/kT]$, and for Si $n_i = 1.5\mathrm{x}10^{10}$ cm^{-3},

$$E_F - E_i = kT \ln\left(\frac{n_0}{n_i}\right) = 0.0259 \ln\left(\frac{10^{14}}{1.5 \cdot 10^{10}}\right) = 0.228\mathrm{eV}.$$

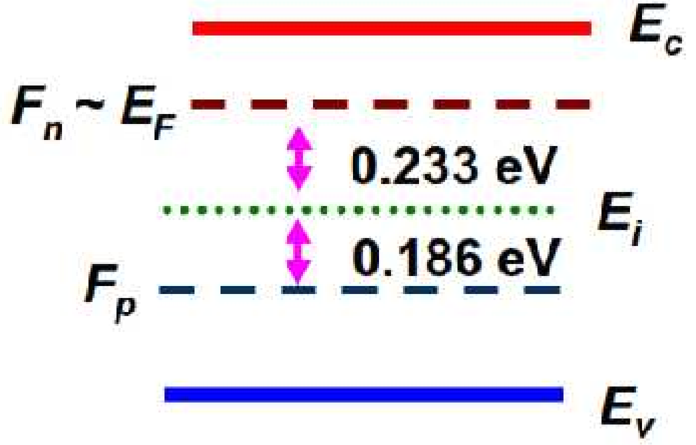
$\delta n = \delta p = g_{op}\tau_n$, thus

$$n = n_0 + \delta n = n_0 + g_{op}\tau_n = 10^{14} + 10^{13} \cdot 2 = 1.2 \cdot 10^{14}\,\mathrm{cm}^{-3}.$$

Since $p_0 << \delta p$, $p \approx \delta p = 2 \cdot 10^{13}\,\mathrm{cm}^{-3}$.

$$F_n - E_i = kT \ln\left(\frac{n}{n_i}\right) = 0.0259 \cdot \ln\left(\frac{1.2 \cdot 10^{14}}{1.5 \cdot 10^{10}}\right) = 0.233\mathrm{eV}$$

$$E_i - F_p = kT \ln\left(\frac{p}{n_i}\right) = 0.0259 \cdot \ln\left(\frac{2 \cdot 10^{13}}{1.5 \cdot 10^{10}}\right) = 0.186\mathrm{eV}.$$



Photoconductive Devices

Photoconductors are light sensitive devices. There are many applications for devices which changes their resistance when exposed to light.

Photoconductor ➡

- Light Detectors
- Moving-object counters
- Burglar Alarms

Light generated Steady state excess carrier → $\Delta n = g_{op}\tau_n \qquad \Delta p = g_{op}\tau_p$

→ Change in conductivity is

$$\Delta\sigma = q\Delta n\mu_n + q\Delta p\mu_p$$

$$\Delta\sigma = qg_{op}\left(\tau_n\mu_n + \tau_p\mu_p\right)$$

Important property is

The time response of a photoconductive cell ⬅

Parameters limiting time response are

- The recombination time
- The degree of carrier trapping
- The time required for carriers to drift through the device in an electric field.

These parameters can be optimized to have optimized time response for given application →

Optimization parameters: sensitivity, response time, dark resistance, device geometry, etc

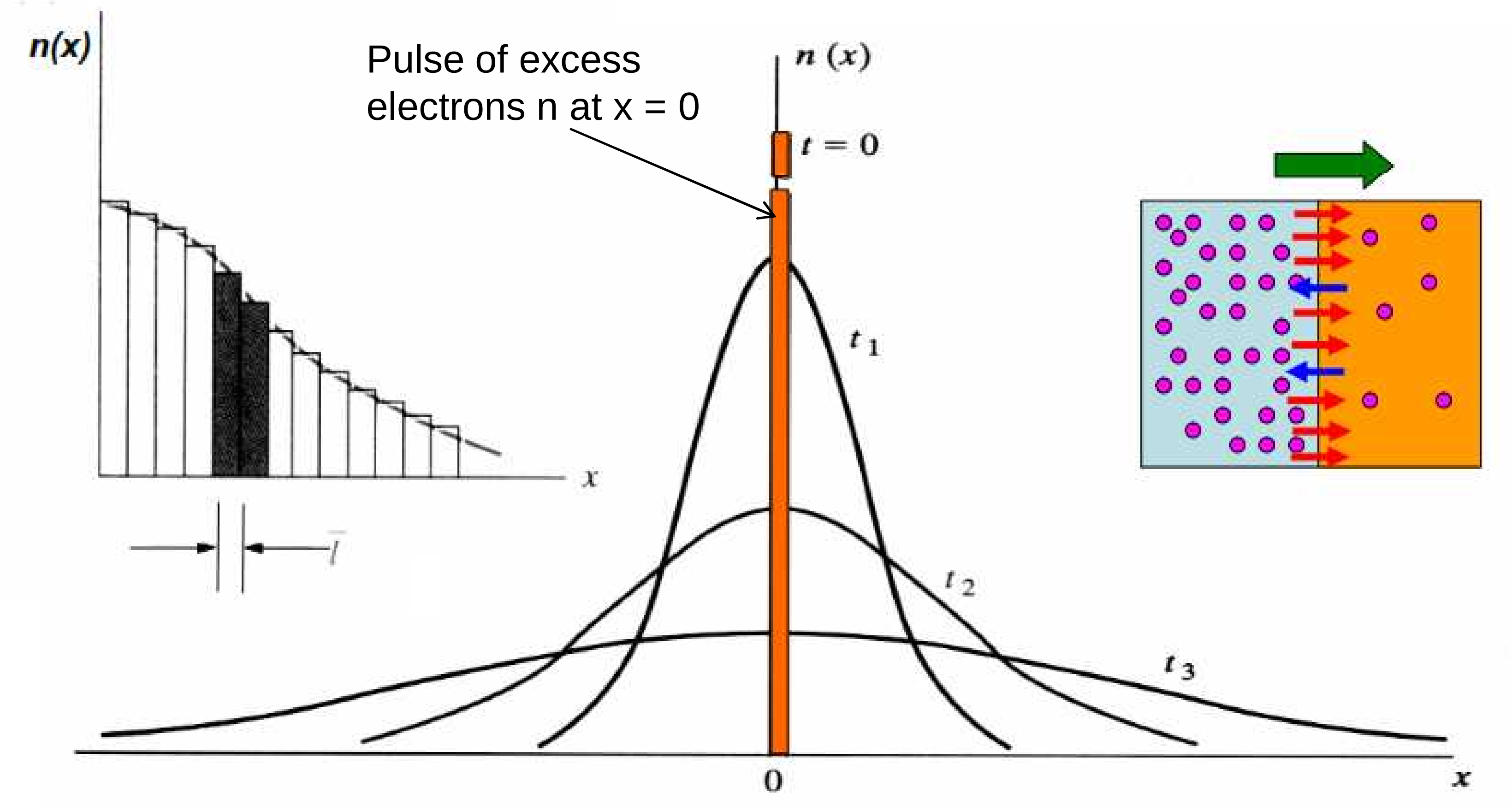
Diffusion Process (no Recombination) (1)

When excess carriers are created in a semiconductor by illumination →

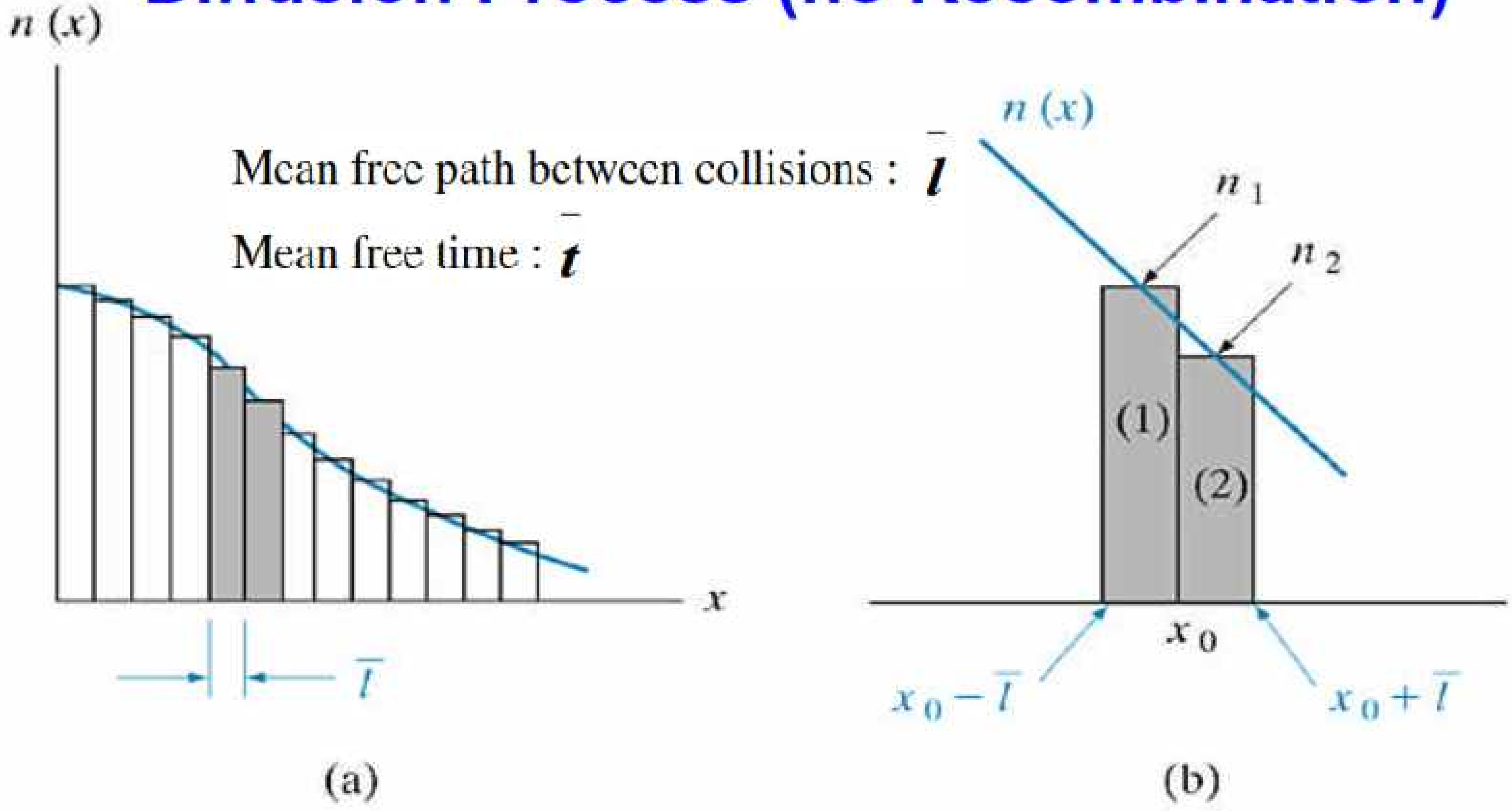
(1) Non-uniform distribution of excess carriers in a semiconductor

(2) Random thermal motion of carriers due to carriers gradient in semiconductor

→ Diffusion of carriers (n and p) from high concentration to low concentration


Pulse of excess electrons n at x = 0

Diffusion Process (no Recombination)



Mean free path between collisions : $\bar{l}$

Mean free time : $\bar{t}$

An arbitrary electron concentration gradient in one dimension: (a) division of $n(x)$ into segments of length equal to a mean free path for the electrons; (b) expanded view of two of the segments centered at x_0.

Half of electrons in 1 will move to 2 and half of electrons in two will move to 1

→ Net # of electrons passing $\boldsymbol{x_0}$ from the left to the right in $\bar{t}$:

$$\frac{1}{2}(n_1 \bar{l} A) - \frac{1}{2}(n_2 \bar{l} A)$$

A = cross-sectional area perpendicular to $\boldsymbol{x}$

Diffusion Process (no Recombination)

Rate of electron flow in +x direction per unit area = electron flux density:

$$\phi_n(x_0) = \frac{\bar{l}}{2\bar{t}}(n_1 - n_2)$$ where

$$n_1 - n_2 = \frac{n(x) - n(x + \Delta x)}{\Delta x}\bar{l}$$ when $\bar{l}$ is very small

x is taken at the center of the segment and Δx = *l*

For Δx →0

$$\phi_n(x_0) = \frac{\bar{l}^2}{2\bar{t}} \lim_{\Delta x \to 0} \frac{n(x) - n(x + \Delta x)}{\Delta x} = -\frac{\bar{l}^2}{2\bar{t}}\frac{dn(x)}{dx}$$ (for very small $\bar{l}$)

Carrier Flux Density

For electrons $$\Phi_n(x) = -D_n \frac{dn(x)}{dx}$$

For holes $$\Phi_p(x) = -D_p \frac{dp(x)}{dx}$$

D_n and D_p are electron and hole Diffusivity

Diffusion Process (no Recombination)

Diffusion current crossing a unit area (current density) = (the particle flux density) x (charge of the carrier)

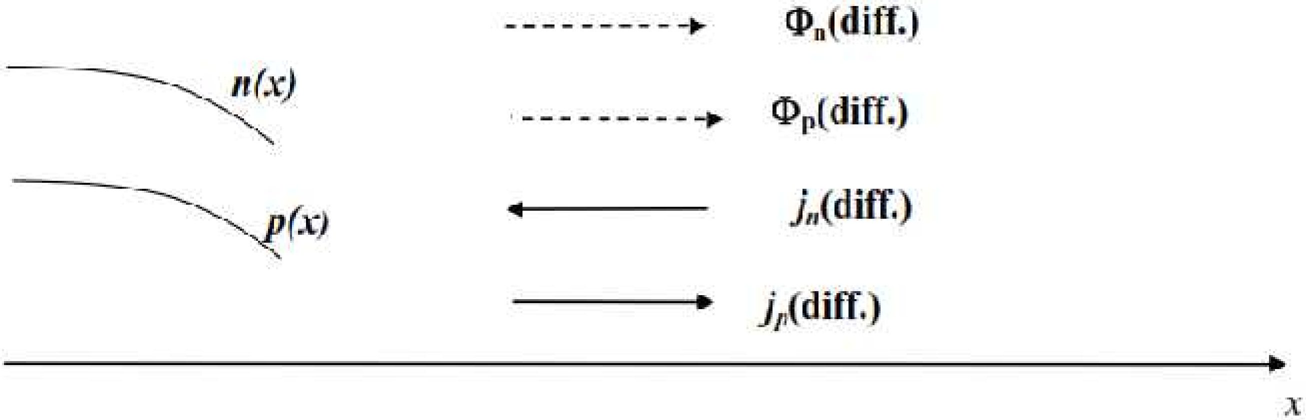
Diffusion Current Density

$$j_n(\text{diff.}) = -(-e)D_n \frac{dn(x)}{dx} = +eD_n \frac{dn(x)}{dx} \rightarrow$$

+ve J → opposite to direction of diffusion (-ve)

$$j_p(\text{diff.}) = -(+e)D_p \frac{dp(x)}{dx} = -eD_p \frac{dp(x)}{dx} \rightarrow$$

-ve J → same direction of diffusion (-ve)



Note that electrons and holes are created at the same location → they move together in the carrier gradient in semiconductor, but resulting current is opposite because of opposite charge of electrons and holes

Diffusion and Drift of Carriers

If an electric field is present in addition to the carrier gradient then, then the total current has a drift as well as a diffusion component and is given by

$$j_n(x) = q\mu_n n(x)\mathcal{E}(x) + qD_n \frac{dn(x)}{dx}$$

Drift term Diffusion term

$$j_p(x) = q\mu_p p(x)\mathcal{E}(x) - qD_p \frac{dp(x)}{dx}$$

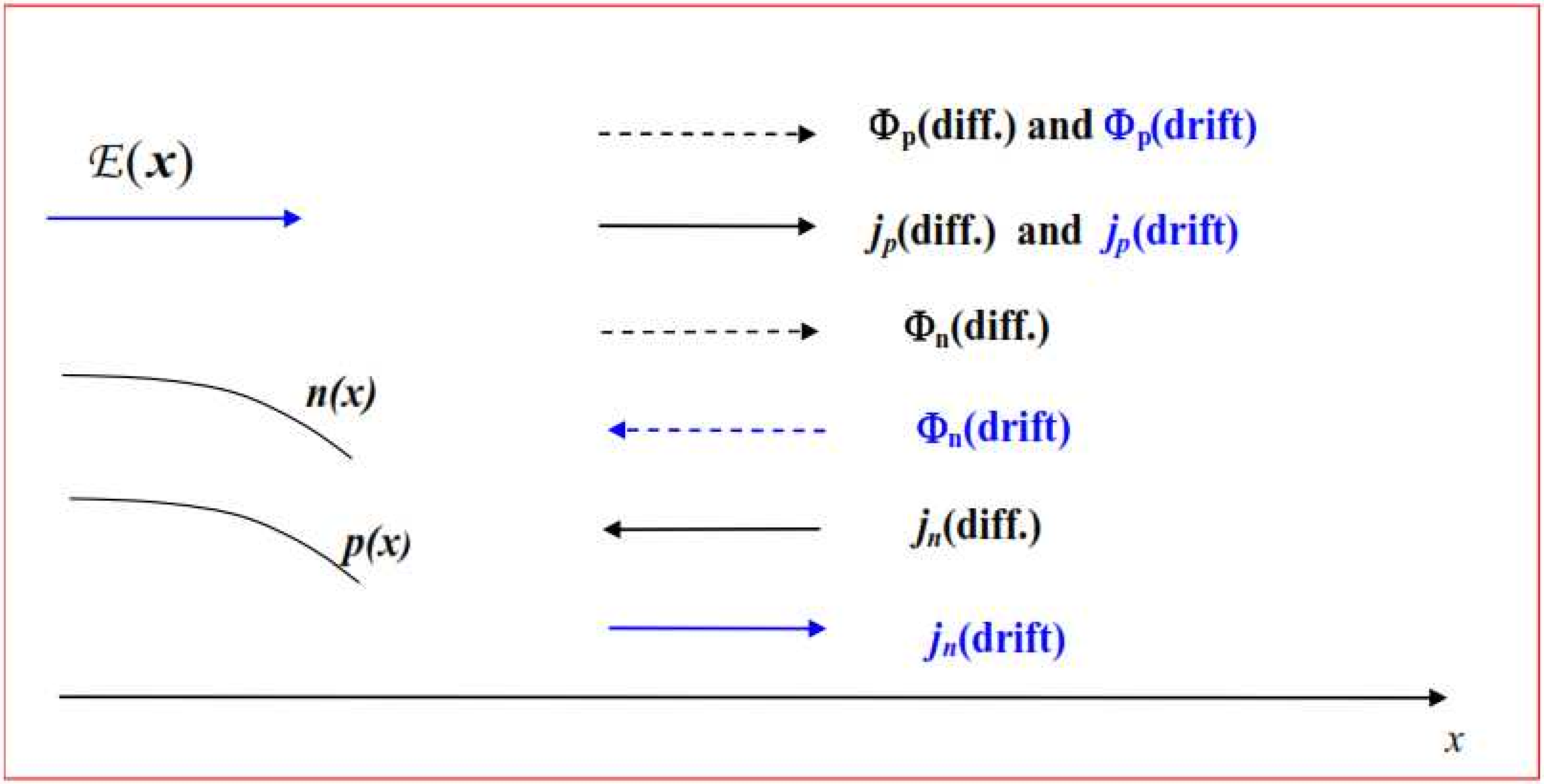
Total current density: $j(x) = j_n(x) + j_p(x)$

Note: Minority carriers can contribute significantly to the current thru diffusion. Since the drift terms are proportional to the carrier concentration, minority carriers seldom provide much drift current.

On the other hand, since the diffusion current is proportional to the gradient of concentration, minority carrier current thru diffusion can sometimes be as large as majority carrier currents.

Diffusion and Drift of Carriers

An imaginary situation to illustrate the drift and diffusion of carriers under applied electric field and a concentration gradient



$\mathcal{E}(x)$

$n(x)$

$p(x)$

Φ_p(diff.) and Φ_p(drift)

j_p(diff.) and j_p(drift)

Φ_n(diff.)

Φ_n(drift)

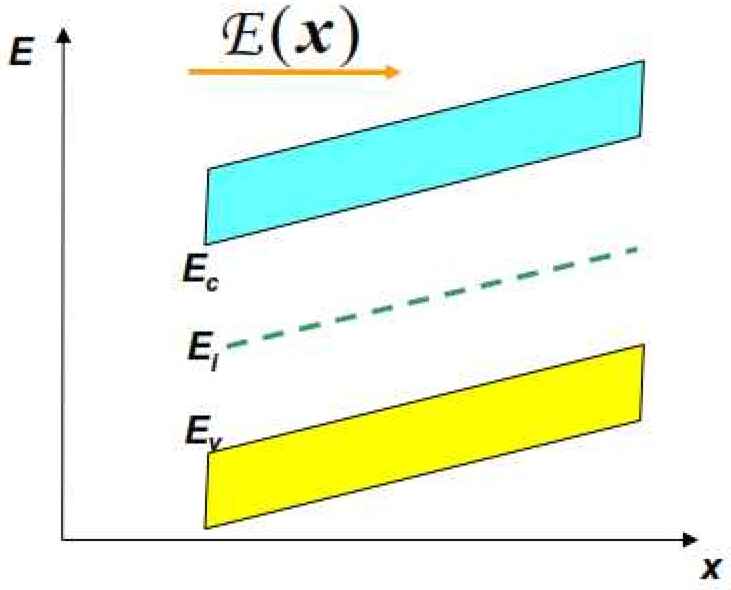
j_n(diff.)

j_n(drift)

x

Diffusion and Drift of Carriers

For electrons, the potential energy increases in the direction of electrical field. The electrostatic potential varies in the opposite direction because electrons have negative charge.



From the definition of electric field *E(x)*, in one-dimensional case,

$$\mathcal{E}(x) = -\frac{dV(x)}{dx}$$

V is electrostatic potential

Choosing E_i (band energy) as a convenient reference, $V(x) = E_i(x)/(-q)$

$$\mathcal{E}(x) = -\frac{dV(x)}{dx} = -\frac{d}{dx}\left[\frac{E_i}{(-q)}\right] = \frac{1}{q}\frac{dE_i}{dx}$$

$$\frac{dE_i}{dx} = q\,\mathcal{E}(x)$$

+ve slope in the direction of electric field

Thus the above shown band diagram is valid for the applied electric field

At equilibrium, no current flows in a semiconductor ! Any fluctuation causin diffusion current sets up an $\mathcal{E}(\boldsymbol{x})$ which redistributes carriers by drift.

For hole current at equilibrium,

$$J_p(x) = q\mu_p p(x)\mathcal{E}(\boldsymbol{x}) - qD_p\frac{dp(x)}{dx} = 0$$

$$\mathcal{E}(\boldsymbol{x}) = \frac{D_p}{\mu_p}\frac{1}{p(x)}\frac{dp(x)}{dx} = \frac{D_p}{\mu_p}\frac{1}{p_0(x)}\frac{dp_0(x)}{dx}$$

Using $\boldsymbol{p_0 = n_i e^{(E_i - E_F)/kT}}$ for $p(x)$ based on **Fig. 4-15 :**

$$\mathcal{E}(\boldsymbol{x}) = \frac{D_p}{\mu_p}\frac{1}{kT}\left(\frac{dE_i}{dx} - \frac{dE_F}{dx}\right)$$

Considering $\frac{dE_F}{dx} = 0$ at equilibrium, and also $\frac{dE_i}{dx} = q\mathcal{E}(\boldsymbol{x})$

Obtain the Einstein relation :

$$\frac{D}{\mu} = \frac{kT}{q}$$

Note : $\mathcal{E}(\boldsymbol{x}) = -\frac{dV(x)}{dx} = -\frac{d}{dx}\left[\frac{E_i}{(-q)}\right] = \frac{1}{q}\frac{dE_i}{dx}$

with Einstein relation $\frac{D}{\mu} = \frac{kT}{q}$ One can calculate either D or μ

At room temperature, for electrons or holes, D/μ=0.0259 V

Diffusion Coefficient and Mobility of Carriers for Intrinsic Semiconductors at 300K.

	$D_n (cm^2/s)$	$D_p (cm^2/s)$	$\mu_n (cm^2/V-s)$	$\mu_p (cm^2/V-s)$
Ge	100	50	3900	1900
Si	35	12.5	1350	480
GaAs	220	10	8500	400

Diffusion and Recombination ; The Continuity Equation

Recombination effects must be considered in a description of conduction processes because the recombination can cause a variation in the carrier distribution.

During diffusion of carriers into material, recombination of carriers will occur → diffusion current will be affected due to the change in carrier concentration

Considering a region of area A and thickness Δx as shown → hole diffusion current entering the region will be difference than current leaving that region due to generation recombination rate in this region

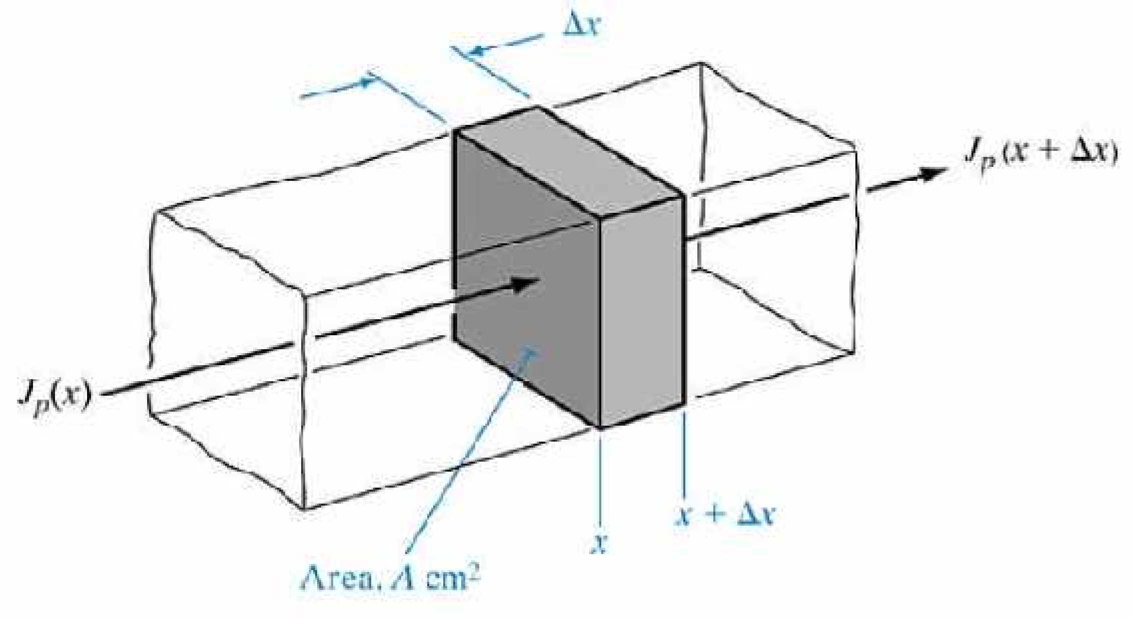


Figure 4—16
Current entering and leaving a volume ΔxA.

Continuity Equations for Holes and Electrons

rate of hole density change in the volume element = change in hole flux density over Δx – recombination rate

$$\left.\frac{\partial p}{\partial t}\right|_{x \to x+\Delta x} = \frac{1}{q}\frac{J_p(x) - J_p(x+\Delta x)}{\Delta x} - \frac{\delta p}{\tau_p}$$

Flux is time rate of
hole flow per unit area

For Δx→ 0, the rate of change in carriers through dx (current continuity equation) is

$$\frac{\partial p(x,t)}{\partial t} = \frac{\partial \delta p}{\partial t} = -\frac{1}{q}\frac{\partial J_p}{\partial x} - \frac{\delta p}{\tau_p} \quad \textit{for holes}$$

$$\frac{\partial n(x,t)}{\partial t} = \frac{\partial \delta n}{\partial t} = +\frac{1}{q}\frac{\partial J_n}{\partial x} - \frac{\delta n}{\tau_n} \quad \textit{for electrons}$$

Since electron charge is -q

Continuity Equations for Holes and Electrons

When the current is carried strictly by diffusion of electron (here, negligible drift) :

$$J_n(diff) = +qD_n \frac{dn(x)}{dx} = qD_n \frac{\partial \delta n}{\partial x}$$

$$\rightarrow \frac{\partial \delta n}{\partial t} = D_n \frac{\partial^2 \delta n}{\partial x^2} - \frac{\delta n}{\tau_n}$$

Diffusion continuity equations

Similarly for holes, $$\frac{\partial \delta p}{\partial t} = D_p \frac{\partial^2 \delta p}{\partial x^2} - \frac{\delta p}{\tau_p}$$

for steady state the time variation is zero $\frac{\partial \delta n}{\partial t} = 0$ and $\frac{\partial \delta p}{\partial t} = 0$

$$\rightarrow \frac{d^2 \delta n}{dx^2} = \frac{\delta n}{D_n \tau_n} = \frac{\delta n}{L_n^2} \qquad L_n = \sqrt{D_n \tau_n}$$

Electron Diffusion length

$$\rightarrow \frac{d^2 \delta p}{dx^2} = \frac{\delta p}{D_p \tau_p} = \frac{\delta p}{L_p^2} \qquad L_p = \sqrt{D_p \tau_p}$$

Hole Diffusion length

Continuity Equations for Holes and Electrons

When the current is carried strictly by diffusion of electron (here, negligible drift) :

$$J_n(diff) = +qD_n \frac{dn(x)}{dx} = qD_n \frac{\partial \delta n}{\partial x}$$

$$\rightarrow \quad \frac{\partial \delta n}{\partial t} = D_n \frac{\partial^2 \delta n}{\partial x^2} - \frac{\delta n}{\tau_n}$$

Diffusion continuity equations

Similarly for holes, $$\frac{\partial \delta p}{\partial t} = D_p \frac{\partial^2 \delta p}{\partial x^2} - \frac{\delta p}{\tau_p}$$

for steady state the time variation is zero $\frac{\partial \delta n}{\partial t} = 0$ and $\frac{\partial \delta p}{\partial t} = 0$

$$\rightarrow \quad \frac{d^2 \delta n}{dx^2} = \frac{\delta n}{D_n \tau_n} = \frac{\delta n}{L_n^2}$$

$$\rightarrow \quad \frac{d^2 \delta p}{dx^2} = \frac{\delta p}{D_p \tau_p} = \frac{\delta p}{L_p^2}$$

$$L_n = \sqrt{D_n \tau_n}$$ Electron Diffusion length

L_n and L_p are Minority carrier diffusion length

$$L_p = \sqrt{D_p \tau_p}$$ Hole Diffusion length

Diffusion length L is average distance that carrier can diffuse before recombination

Steady State Carrier Injection

The general solution of above steady state continuity equation is

$$\delta p(x) = C_1 e^{x/L_p} + C_2 e^{-x/L_p} \qquad \text{(General Solution)}$$

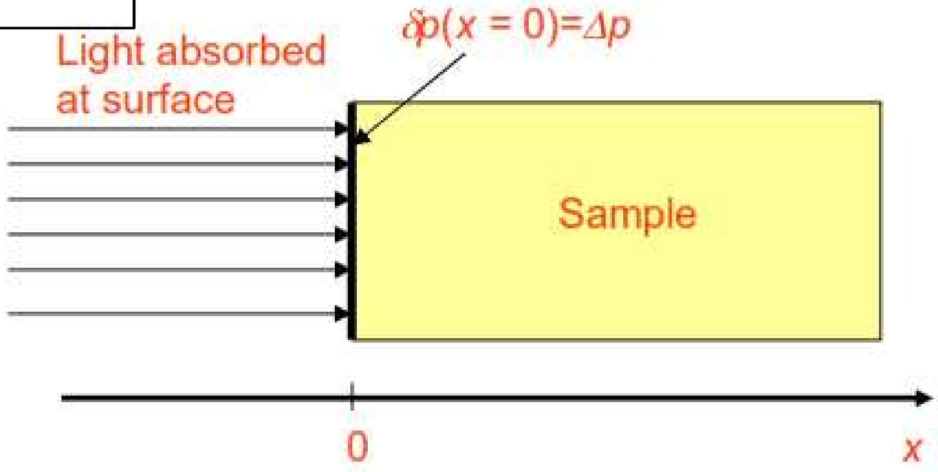
Using boundary conditions

$\delta p(x \rightarrow \infty) = 0 \rightarrow C_1 = 0$. At the injection point of hole ($x = 0$), the hole concentration $\delta p(x=0) = \Delta p \rightarrow C_2 = \Delta p$

Hence $\boxed{\delta p(x) = \Delta p e^{-x/L_p}}$

→ Hole density with x becomes

$$p(x) = p_0 + \Delta p e^{-x/L_p}$$

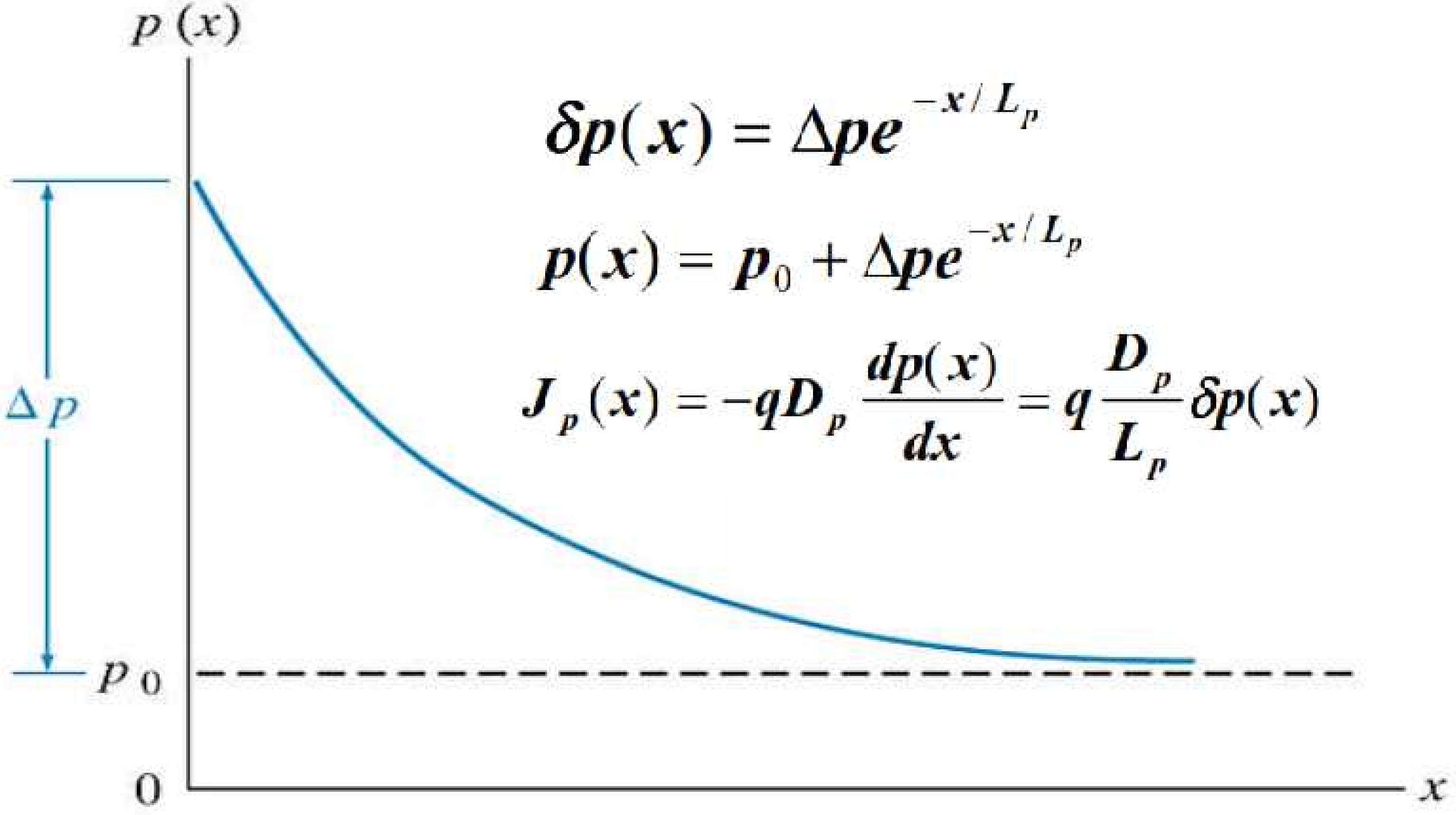


→ The current density

$$J_p(x) = -qD_p \frac{dp(x)}{dx} = q\frac{D_p}{L_p}\delta p(x)$$

Steady State Carrier Injection

Injected excess hole conc. decreases exponentially in *x* due to recombination & the diffusion length (L_p) represents the distance at which **the excess hole distribution is reduced to *1/e* of its value at the injection point of hole** (Δp) .



$$\delta p(x) = \Delta p e^{-x/L_p}$$

$$p(x) = p_0 + \Delta p e^{-x/L_p}$$

$$J_p(x) = -qD_p \frac{dp(x)}{dx} = q\frac{D_p}{L_p}\delta p(x)$$

EXAMPLE 4–5 In a very long p-type Si bar with cross-sectional area = 0.5 cm^2 and N_a = 10^{17} cm^{-3}, we inject holes such that the steady state *excess* hole concentration is 5×10^{16} cm^{-3} at $x = 0$. What is the steady state separation between F_p and E_c at x = 1000 Å? What is the hole current there? How much is the excess stored hole charge? Assume that μ_p = 500 cm^2/V-s and $\tau_p = 10^{-10}$ s.

$$D_p = \frac{kT}{q}\mu_p = 0.0259 \times 500 = 12.95 \text{ cm/s}$$

$$L_p = \sqrt{D_p\tau_p} = 3.6 \times 10^{-5} \text{ cm}$$

$$p = p_0 + \Delta p e^{-\frac{x}{L_p}} = 10^{17} + 5 \times 10^{16} e^{-\frac{10^{-5}}{3.6\times10^{-5}}} = 1.379 \times 10^{17}$$

$$p = n_i e^{(E_i-F_p)/kT} = (1.5 \times 10^{10} \text{ cm}^{-3}) e^{(E_i-F_p)/kT}$$

$$E_i - F_p = \left(\ln\frac{1.379 \times 10^{17}}{1.5 \times 10^{10}}\right)\cdot 0.0259 = 0.415 \text{ eV}$$

$$E_c - F_p = 1.1/2 \text{ eV} + 0.415 \text{ eV} = \mathbf{0.965\ eV}$$

$$I_p = qA\frac{D_p}{L_p}(\Delta p)e^{-\frac{x}{L_p}} = 1.6 \times 10^{-19} \times 0.5 \times \frac{12.95}{3.6 \times 10^{-5}} \times 5 \times 10^{16} e^{-\frac{10^{-5}}{3.6\times10^{-5}}}$$
$$= \mathbf{1.09 \times 10^3\ A}$$

Charge stored will be in a volume contained in a diffusion length (L_pA)

→ $$Q_p = qA(\Delta p)L_p = 1.6 \times 10^{-19}(0.5)(5 \times 10^{16})(3.6 \times 10^{-5}) = \mathbf{1.44 \times 10^{-7}\ C}$$

Haynes – Shockley Experiment

- Demonstration of the drift and diffusion of the minority carriers (J. R. Haynes and W. Shockley, 1959, Bell Lab)

\- **Independent measurement of minority carrier mobility (μ) and diffusion coefficient (D)**

\- **A pulse of holes is created in an n-type bar that contains an electric field**

\- As the pulse drifts in the field and spreads out by diffusion, the excess hole concentration is monitored at some point down the semiconductor bar

\- The time required for the hole to drift a given distance in the field gives a measure of the **mobility**; and the spreading of the pulse during a given time is used to calculate the **diffusion coefficient**

Haynes – Shockley Experiment

Measurement of the minority carrier's mobility and diffusion coefficient

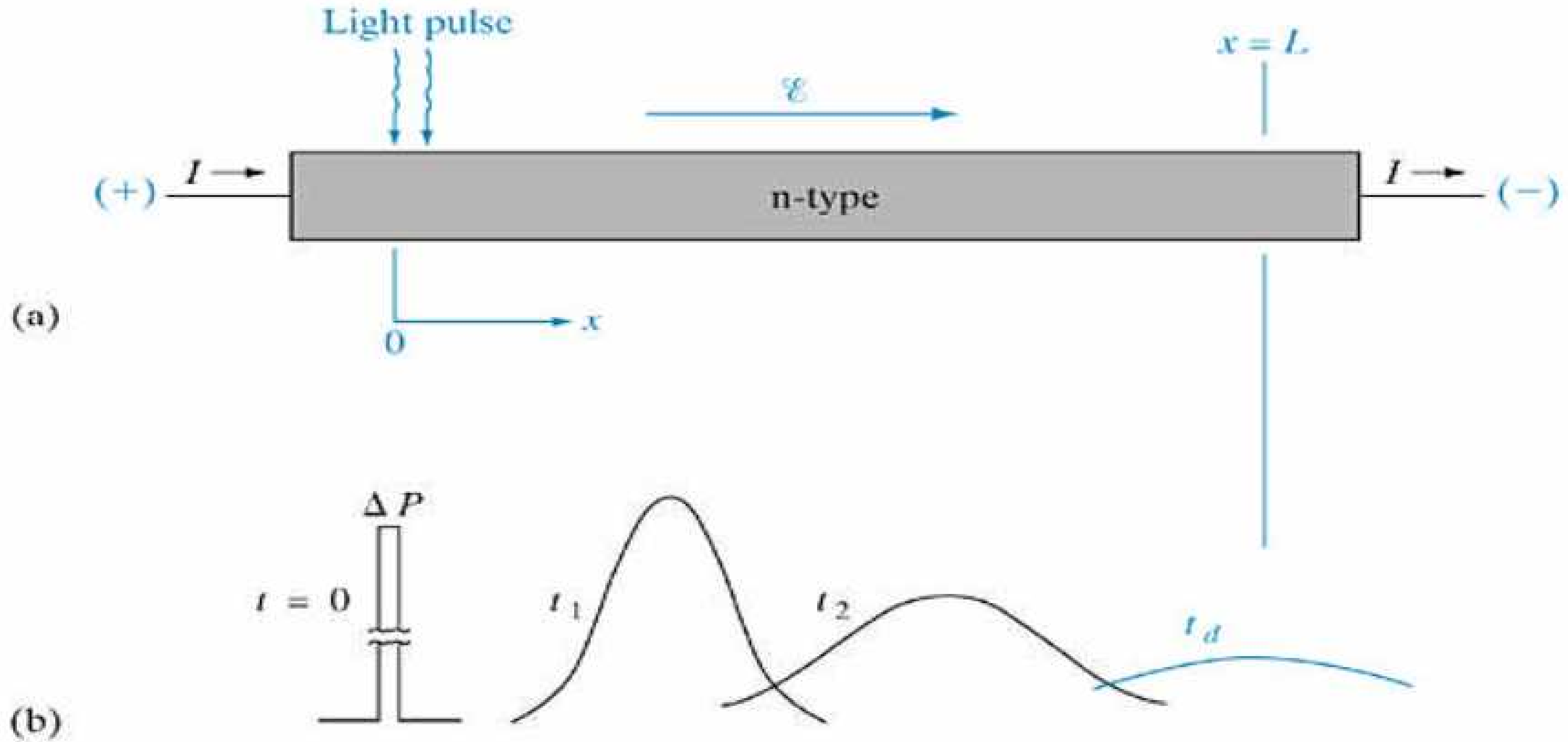


Figure 4—18
Drift and diffusion of a hole pulse in an n-type bar: (a) sample geometry; (b) position and shape of the pulse for several times during its drift down the bar.

Basic principles of the Haynes-Shockley experiment !

(1) A pulse of holes is created in an ***n-type*** bar of length $\boldsymbol{L}$ that contains an electric field (Fig. 4-18).

(2) As the pulse **drifts in the electric field** and **spreads out by diffusion**, the excess hole concentration is monitored at some point down the bar.

(3) The drift time ($\boldsymbol{t_d}$) required for the holes to drift a given distance in the field gives a measure of the mobility ($\boldsymbol{\mu_p}$) and drift velocity ($\boldsymbol{V_d}$).

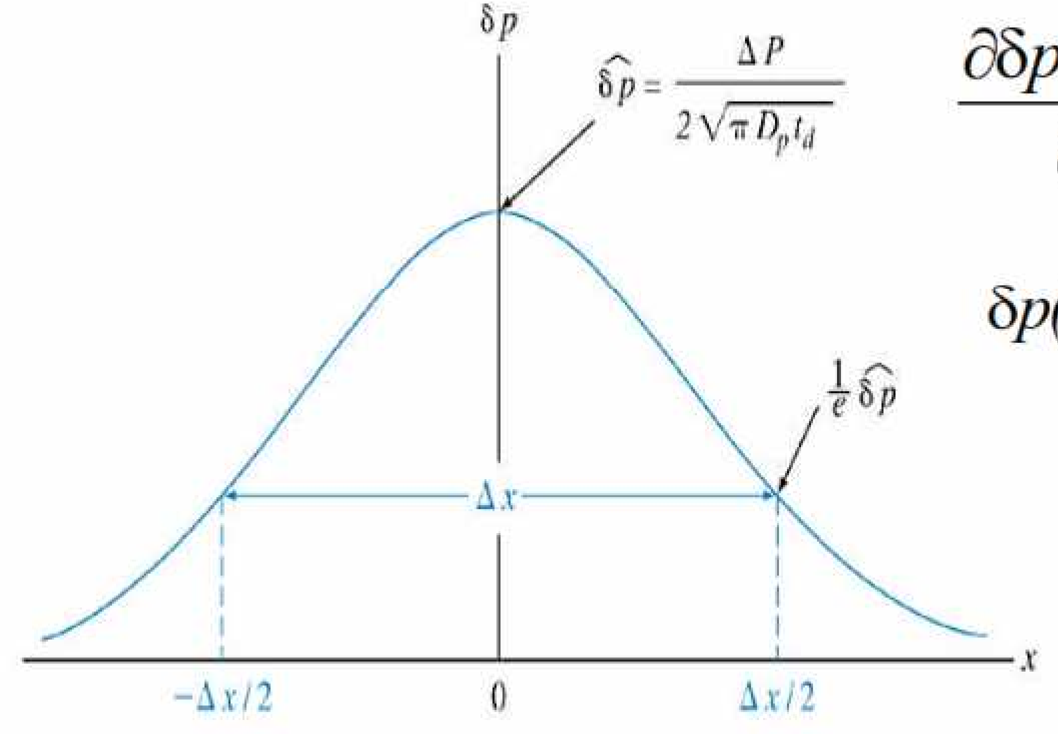
(4) The spreading of the pulse during a given time is used to calculate the diffusion coefficient ($\boldsymbol{D_p}$).

$$V_d = \frac{L}{t_d} \qquad \mu_p = \frac{V_d}{\varepsilon}$$

Note that the electric field $\varepsilon = \Delta V/d$, where ΔV is potential difference and d is the bar length

Haynes – Shockley Experiment

Assume that the hole profile has a **gaussian distribution** in the time-dependent diffusion.



$$\frac{\partial \delta p(x,t)}{\partial t} = D_p \frac{\partial^2 \delta p(x,t)}{\partial x^2}$$

$$\delta p(x,t) = \left[\frac{\Delta P}{2\sqrt{\pi D_p t}}\right] e^{-x^2/4D_p t}$$

$$\delta p = \left[\frac{\Delta P}{2\sqrt{\pi D_p t}}\right]$$

Figure 4—19
Calculation of D_p from the shape of the δp distribution after time t_d. No drift or recombination is included

The factor in brackets indicates that the **peak value** of the pulse (at $x = 0$) decreases with time, and the exponential factor predicts the spread of the pulse in the positive and negative ***x-directions*** (Fig. 4-19).

Designate **the peak value of the pulse** as δp at any time (t_d) and calculate D_p from the value of δp at some point x. The most convenient choice is the point $\Delta x/2$, at which δp is down by $1/e$ of its peak value δp.

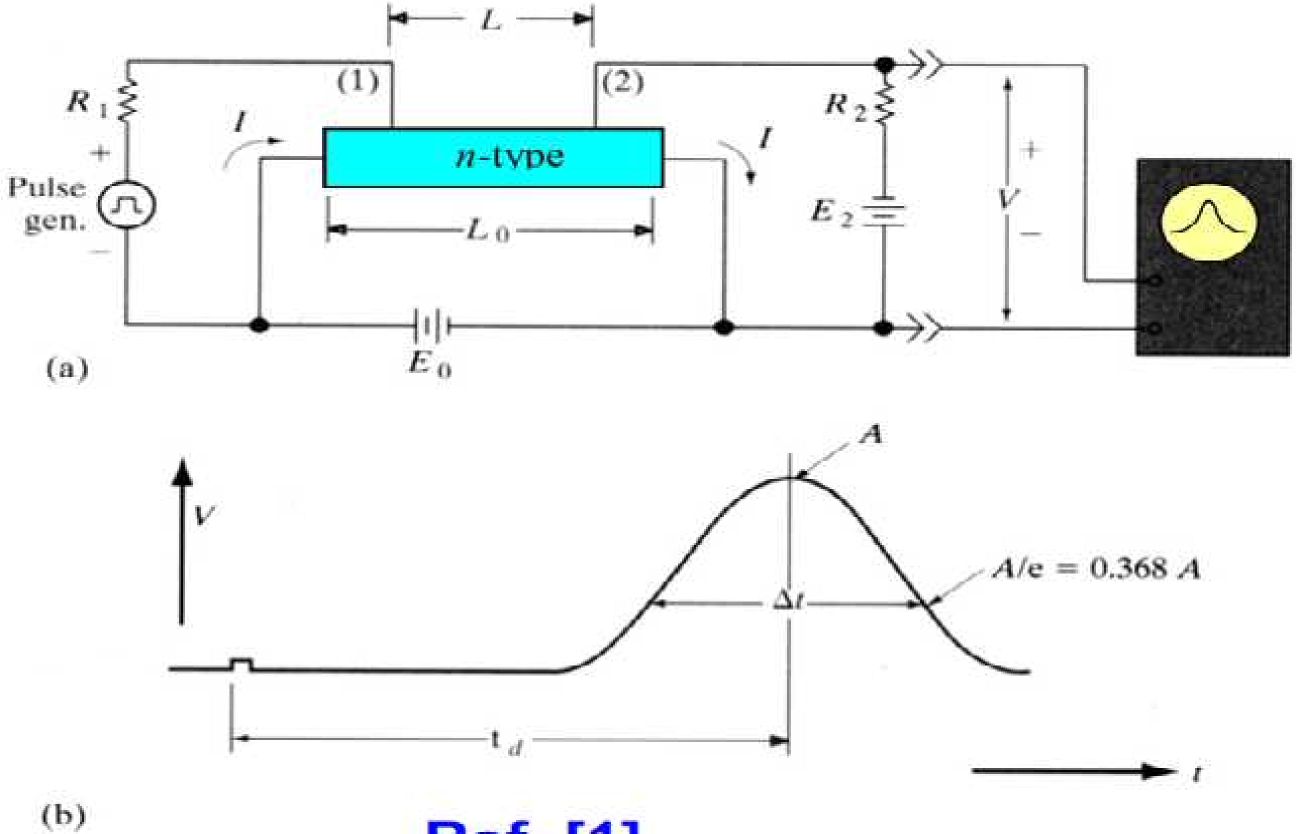
$$\delta p = \left[\frac{\Delta P}{2\sqrt{\pi D_p t}} \right]$$

$$e^{-1} \delta p = \delta p e^{-(\Delta x/2)^2 / 4 D_p t_d}$$

$$\boxed{D_p = \frac{(\Delta x)^2}{16 t_d}}$$

Haynes – Shockley Experiment

Since Δx can not be directly measured, we use an experimental setup, allowing for the display of the pulse on an oscilloscope as the carriers pass under a detector. The measured quantity is the **pulse width (Δt)** displayed on the oscilloscope in time. The **Δt** is related to Δx by the drift velocity (V_d) , as the pulse starts from the injection point (1) and drifts to the detector point (2).



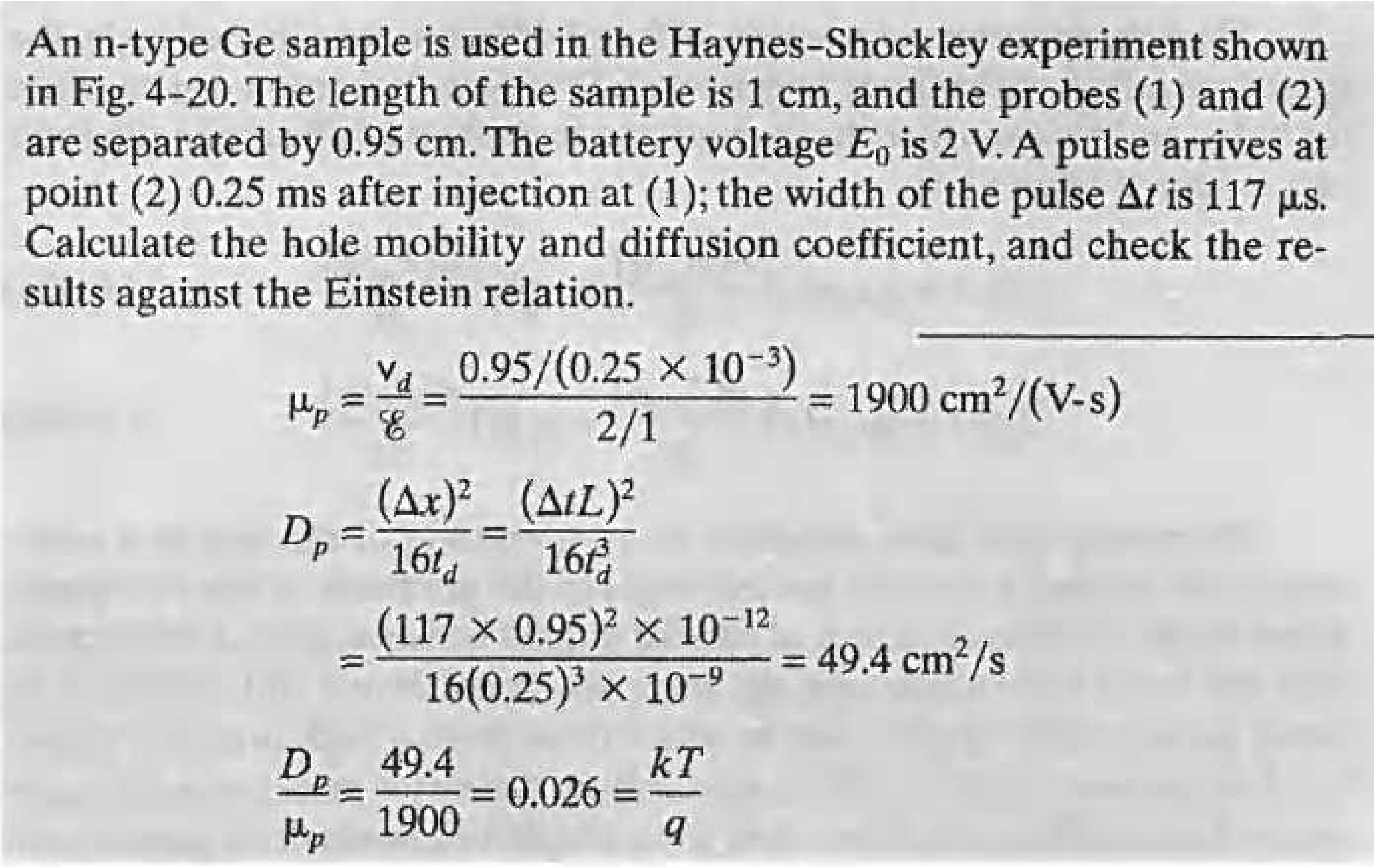
$$\Delta x = \Delta t V_d$$

$$\Delta x = \Delta t \frac{L}{t_d}$$

$$\frac{\mu_p}{D_p} = \frac{kT}{q}$$

Ref. [1]

Figure 4—20
The Haynes–Shockley experiment: (a) circuit schematic; (b) typical trace on the oscilloscope screen.



An n-type Ge sample is used in the Haynes–Shockley experiment shown in Fig. 4–20. The length of the sample is 1 cm, and the probes (1) and (2) are separated by 0.95 cm. The battery voltage E_0 is 2 V. A pulse arrives at point (2) 0.25 ms after injection at (1); the width of the pulse Δt is 117 μs. Calculate the hole mobility and diffusion coefficient, and check the results against the Einstein relation.

$$\mu_p = \frac{v_d}{\mathscr{E}} = \frac{0.95/(0.25 \times 10^{-3})}{2/1} = 1900 \text{ cm}^2/(\text{V-s})$$

$$D_p = \frac{(\Delta x)^2}{16 t_d} = \frac{(\Delta t L)^2}{16 t_d^3}$$

$$= \frac{(117 \times 0.95)^2 \times 10^{-12}}{16(0.25)^3 \times 10^{-9}} = 49.4 \text{ cm}^2/\text{s}$$

$$\frac{D_p}{\mu_p} = \frac{49.4}{1900} = 0.026 = \frac{kT}{q}$$

HW: 4.2, 4.6, 4.10, 4.13

Gradients in the Quasi-Fermi Levels

Not like Fermi level at equilibrium, the non-equilibrium Quasi Fermi level is affected by non-equilibrium carrier concentration within the material. Hence, electron current density can be written in term of quasi Fermi level as followed:

$$J_n(x) = q\mu_n n(x)\varepsilon(x) + qD_n \frac{dn(x)}{dx} \quad \text{but} \quad \frac{dn(x)}{dx} = \frac{d}{dx}\left[n_i e^{(F_n - E_i)/kT}\right] = \frac{n(x)}{kT}\left(\frac{dF_n}{dx} - \frac{dE_i}{dx}\right)$$

$$J_n(x) = q\mu_n n(x)\varepsilon(x) + \mu_n n(x)\left[\frac{dF_n}{dx} - \frac{dE_i}{dx}\right]$$

$$= q\mu_n n(x)\varepsilon(x) + \mu_n n(x)\left[\frac{dF_n}{dx} - q\varepsilon(x)\right]$$

$$J_n(x) = \mu_n n(x)\frac{dF_n}{dx} = q\mu_n n(x)\frac{d(F_n/q)}{dx} = \sigma_n(x)\frac{d(F_n/q)}{dx}$$

For holes, the expressions are similar.

Modified Ohm's Law:

$$\boxed{\begin{aligned} j_n(x) &= q\mu_n n(x)\frac{d(F_n/q)}{dx} = \sigma_n(x)\frac{d(F_n/q)}{dx} \\ j_p(x) &= q\mu_p p(x)\frac{d(F_p/q)}{dx} = \sigma_p(x)\frac{d(F_p/q)}{dx} \end{aligned}}$$

Note that this is a the most general expression showing that current can be caused by change in concentration or the electric field (both affects the quasi Fermi Level)